

- 1) Boolean algebra was formulated by English Mathematician George Boole in
(a) 1857 (b) 1853 (c) 1854 (d) 1847

2) If Z is a Boolean variable then its value can be
(a) 0 - 9 (b) 0 - 7 (c) 0 & 1 (d) 0 or 1

3) Which of the following are basic logical operators in Boolean algebra?
(a) AND (b) OR (c) NOT (d) All

4) The operator that gives complement of the given value is called
(a) AND (b) OR (c) NOT (d) All

5) If $A = 1$, $B = 0$ then $A \text{ And } B = ?$
(a) 0 (b) 10 (c) 11 (d) 1

6) Boolean Algebra is the _____ of logic
(a) Mathematics (b) Solution (c) Algebra (d) Geometry

7) _____ uses symbols to represents logical statements instead of words.
(a) Algebra (b) Geometry (c) Boolean Algebra (d) Digital data

8) If $A = 1$, $B = 1$, $C = 0$ then $A \cdot B \cdot C = ?$
(a) 0 (b) 10 (c) 11 (d) 1

9) If $A = 1$, $B = 1$, $C = 0$ then $A + B + C = ?$
(a) 0 (b) 10 (c) 01 (d) 1

10) If $A = 1$, $B = 0$, $C = 1$ then $X = A + B + C$
(a) 0 (b) 101 (c) 1 (d) 10

11) $A + B = B + A$ and $A \cdot B = B \cdot A$ represents which of the followings.
(a) Existence of identity (b) Commutative law
(c) Associative law (d) Idempotent law

12) Boolean Algebra was formulated by the
(a) Charles Babbage (b) Napir (c) Pascal (d) George Boole

13) According to distributive law $A \cdot (B + C) = ?$
(a) $A + (B \cdot C)$ (b) $(A + B) \cdot (A + C)$ (c) $A \cdot B + A \cdot C$ (d) $(A + B) + C$

14) According to Existence of identity element $A \cdot 1 = ?$
(a) 1A (b) 0 (c) A (d) 1

15) As per Existence of inverse $A + A = ?$
(a) 0 (b) 2A (c) 0 (d) 1

16) Computer chips are made up of
(a) Transmitters (b) Transistors (c) Circuits (d) None

17) According to inverse the output will be 0 if
(a) $A + A$ (b) $A \cdot A$ (c) $1 + A$ (d) $0 + A$

18) Boolean Algebra consists of
(a) Variables (b) Constant (c) Both a & b (d) Nothing

19) AND operator represent
(a) . (b) * (c) / (d) “ ”

20) In Boolean Algebra each variable at one time can taken how many values?
(a) 1 (b) 2 (c) 3 (d) 4

21) Which is called an unary operator?
(a) NOT (b) AND (c) OR (d) All

22) All electronic devices consist of circuits of
(a) Buttons (b) Cables (c) Elements (d) Switches

Chapter 6

BOOLEAN ALGEBRA

ANSWER KEY

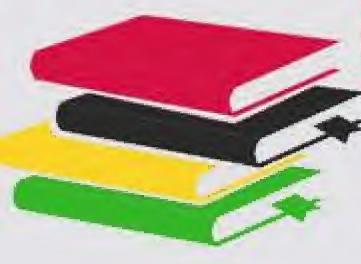
1	A	11	B	21	A	31	A	41	C
2	D	12	D	22	D	32	C	42	B
3	D	13	C	23	C	33	D	43	C
4	C	14	C	24	B	34	A	44	C
5	A	15	D	25	D	35	B	45	B
6	C	16	C	26	A	36	C	46	B
7	C	17	B	27	B	37	D	47	C
8	A	18	C	28	C	38	B	48	C
9	D	19	A	29	C	39	B	49	C
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SHORT QUESTIONS**Q1: What is Boolean Algebra?****BOOLEAN ALGEBRA**

In 1847, a British Mathematician, George Boole, introduced the algebra of binary numbers. This is known as Boolean algebra.

Q2: What is Proposition?**PROPOSITION**

The statements that return result true or false are called propositions. Sentence "What is your name?" is not a proposition because its answer is not in the form of yes (true) or no (false) form. On the other hand the sentence "Are you Male?" is a proposition because its answer is in the form of Yes No.

Q3: What are Boolean Constants?**BOOLEAN CONSTANTS**

The quantities that may not change their values are called constants. The two binary values 0 and 1 are called Boolean constants. These values may be used in Boolean expressions. If $B = \{0, 1\}$ then 0 and 1 are the Boolean constants.

Q4: What are Boolean Variables?**BOOLEAN VARIABLES**

The variables used in Boolean algebra are called Boolean variables, Boolean variable may have value 0 or 1 and it is denoted by single letter such as V, 'y', 'a', 'b' etc.

Q5: What is AND Operation?**AND OPERATION**

The process to perform logical multiplication of two binary variables or numbers in Boolean algebra is called AND Operation. The AND operator is used for AND Operation. This operator is represented by a symbol • (Dot) or by absence of any symbol.

Q6: What is OR Operation?**OR OPERATION**

The process to perform logical addition of two binary variables or numbers in Boolean algebra is called OR operation. The OR operator is used for OR operation. This operator is represented by a symbol + sign.

Q7: What is NOT Operation?**NOT OPERATION**

The process to take the complement of a binary variable or number is called logical NOT operation. In this operation, the prime or bar sign is placed over the variables such as: X or \bar{X} .

Q8: What is Truth Table?**TRUTH TABLE**

A table that represents the output from different combinations of input variables is called truth table.

Q9: What is Boolean Expression?**BOOLEAN EXPRESSION**

An expression formed with binary variables, constants, Boolean operators as well as parentheses is known as Boolean Expression. It is a logical statement, which gives result either true or false.

Q10: What is Identity Element?**IDENTITY ELEMENT**

There is an identity element '1' with respect to AND (.) and an identity element '0' with respect to OR such that for 'x':

- a) $x + 0 = x$
- b) $x \cdot 1 = x$

Q11: What is Distributive law?**DISTRIBUTIVE LAW**

The OR (+) operation is distributive over AND (.) and AND (.) operation is distributive over OR (+) operation. Suppose x, y and z are members of set B then according to distributive laws.

- (a) $x \cdot (y + z) = x \cdot y + x \cdot z$
- (b) $x + (y \cdot z) = (x + y) \cdot (x + z)$

Q12: What is Duality Principle?**DUALITY PRINCIPLE**

If the binary operators (such as '+' and '!') and identity elements (such as 1 and 0) of one part of axiom is interchanged then other part of axiom is automatically obtained. This is an important property of Boolean algebra. This property is called the duality principle.

Q13: What is De-Morgan's Law?**DE-MORGAN'S LAW**

The complement of addition of two numbers is equal to the product of their complements. Similarly the complement of product of two numbers is equal to the sum of their complements.

If 'x' and 'y' are two Boolean variables then according to De Morgan's law.

- (a) $x + y = x \cdot y$
- (b) $x \cdot y = x + y$

Q14: What is Boolean function?**BOOLEAN FUNCTION**

A Boolean function is an expression formed with binary variables, Boolean operators, parenthesis and an equal sign. The Boolean variables may be equal to either 0 or 1. The value returned by Boolean function is also equal to either 0 or 1.

Q15: What is Literal?**LITERAL**

Suppose two variables 'x' and 'y' are used in a Boolean function. Each variable may appear in the function in two forms, i.e. it may appear in the complement form or without complement form. Each of these forms is called a literal. Each literal represents one input to the Boolean function.

Q16: What is Minterm?**MINTERM**

If a product of Boolean variables is 1, it is known as Minterm. It is also known as standard product.

Q17: What is Maxterm?**MAXTERM**

If the sum of Boolean variables is 0, it is known as Maxterm. It is also known as standard sum.

Q18: What is Karnaugh Map?

KARNAUGH MAP

Karnaugh Map is also referred to as K-map. It provides an efficient way to solve/simplify Boolean functions. It is a tool used to transform a truth table of expression into a simplified logic circuit.

Q19: What is Axiom?

AXIOM

Boolean algebra consists of certain basic postulates and theorems that are used in the simplification of Boolean expressions for designing electronic circuits. These basic postulates are called axioms.

Q21: What are the disadvantages of using Boolean Algebraic laws?

DISADVANTAGES OF USING BOOLEAN ALGEBRAIC LAWS.

Following are some disadvantages of using Boolean algebraic laws for simplification of Boolean expressions.

1. It is difficult to write a computer program that can use these laws to simplify a given Boolean function.
2. This process may not give the best-simplified function.
3. A Boolean function is needed for this process to work. But in most engineering applications we do not have the actual Boolean function and only have the truth table of the required function.

Q20: What are the advantages of Karnaugh Map?

ADVANTAGES

Some advantages of this method of simplification are given below

1. This method is very easy to follow
2. This is a systematic process. It always leads to a single minimal solution

DISADVANTAGES

A disadvantage of this system is that it is not scalable. This means that this system works very well for less variables but becomes complex for higher number of variables.

LONG QUESTION

Q1: Write a note on logic gates.

LOGIC GATES

The electronic circuits inside the computer that get one or more input signals and produce standard output signals are called logic gates. All types of operations inside the computer are carried out by logic gates. The input and output signals are in the form of digital signals. The logic gates are also known as logic circuits, switching circuits or digital circuits.

The most important basic gates are

1. OR GATE
2. AND GATE
3. NOT GATE

OR-GATE

An OR gate is a very simple electronic circuit. It gets two or more input signals and gives a single output. If any of the input signals is 1, the output will be 1. If all input signals are 0, then the output will also be 0.

The OR gates are used inside the computer to perform logical addition. The symbol '+' is used to represent logical addition. It is also known as OR operator

SYMBOL FOR OR-GATE

The symbol for OR-gate is shown below. In the symbol, A, B and C are the inputs and O is the output.

**AND-GATE**

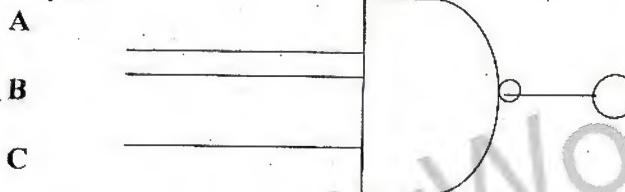
An AND-gate is an electronic circuit used inside the computer. Like an OR-gate, two or more inputs may be given but it gives only one output.

In an AND-gate, if all inputs are 1, the output will be 1, otherwise the output will be 0.

The AND-gate circuits perform logical multiplication. The symbol [.] (dot) is used to represent logical multiplication. It is also known as AND operator.

SYMBOL FOR AND-GATE

The symbol for an AND-gate is given below. A, B and C are the inputs and O is the output.

**NOT-GATE**

A NOT-gate is a simple electronic circuit inside the computer. In this type of circuits, only one input signal is given and one output is received. The NOT-gate gives an output signal which is the opposite of the given input signal. The NOT-gate inverts the input signal. A NOT-gate is also known as an inverter.

SYMBOL FOR NOT-GATE

The symbol for a NOT-gate is shown in the figure below.

**Exercise**

- State and prove the De Morgan's Law for the Boolean Algebra.
The complement of addition of two numbers is equal to the product of their complements. If x and y are two Boolean variable, then

$$X + Y = \bar{X} \cdot \bar{Y}$$

X	Y	\bar{X}	\bar{Y}	$X+Y$	$\bar{X} \cdot \bar{Y}$	$\bar{X} \cdot \bar{Y}$
0	0	1	1	0	1	1
0	1	1	0	1	0	0
1	0	0	1	1	0	0
1	1	0	0	0	0	0

X.Y X + Y

X	Y	\bar{X}	\bar{Y}	X.Y	$X+Y$	$\bar{X}+\bar{Y}$
0	0	1	1	0	1	1
0	1	1	0	0	1	1
1	0	0	1	0	1	1
1	1	0	0	1	0	0

2. If X and Y are Boolean variables then prove the following identities by using the truth table.

(a) $X + Y$

X	Y	\bar{X}	\bar{Y}	$X+Y$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	1
1	1	0	0	0

(a) $X + X \cdot Y = X$

X	Y	$X \cdot Y$	$X + X \cdot Y$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

(a) $X \cdot (X + Y) = X$

X	Y	$X + Y$	$X \cdot (X + Y)$
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

(b) $X + 1 = 1$

X	$X + 1 = 1$
0	1
1	1

(c) $X \cdot 0 = 0$

X	$X \cdot 0 = 0$
0	0
1	0

3. Make truth table of the following functions

(a) $f(x, y) = X \cdot Y + \bar{X} \cdot Y$

X	Y	\bar{X}	$X \cdot Y$	$\bar{X} \cdot Y$	$X \cdot Y + \bar{X} \cdot Y$
0	0	1	0	0	0
0	1	1	0	1	1
1	0	0	0	0	0
1	1	0	1	0	1

(b) $f(x, y) = X \cdot \bar{Y} + \bar{X} \cdot Y$

X	Y	\bar{X}	\bar{Y}	$X \cdot Y$	$\bar{X} \cdot Y$	$X \cdot \bar{Y} + \bar{X} \cdot Y$
0	0	1	1	0	0	0
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	0	0	0	0

1. Calculate the value of the following Boolean functions at the given values of X, Y and Z

(a) $X \cdot Y + X \cdot Z + \bar{X} \cdot Y$ for $X = 0, Y = 1$ and $Z = 0$

$$X = 0 \quad \bar{X} = 1$$

$$Y = 1 \quad \bar{Y} = 0$$

$$Z = 0 \quad \bar{Z} = 1$$

$$X \cdot Y + X \cdot \bar{Z} + \bar{X} \cdot \bar{Y}$$

$$1 \cdot 1 + 1 \cdot 1 + 0 \cdot 0$$

$$1 + 1 + 0$$

$$1$$

(b) $(\bar{X} + Y) \cdot X + (\bar{Y} + Z)$ for $X = 0, Y = 1$ and $Z = 1$

$$X = 0 \quad \bar{X} = 1$$

$$Y = 1 \quad \bar{Y} = 0$$

$$Z = 1 \quad \bar{Z} = 0$$

$$(\bar{X} + Y) \cdot X + (\bar{Y} + Z)$$

$$(1 + 1) \cdot 0 + (0 + 1)$$

$$0 + 1$$

$$1$$

2. Prove the following results and apply the principle of duality to obtain the dual of these results.

(a)

$$X + \bar{X} = X$$

Duality of the above expression

$$X \cdot \bar{X} = X$$

(b)

$$X + 0 = X$$

Duality of the above expression

$$X \cdot 1 = X$$

(c)

$$X + X \cdot Y = \bar{X} + Y$$

Duality of the above expression

$$\bar{X} \cdot X + Y = \bar{X} \cdot Y$$

(d)

$$X \cdot (Y + Z) = (X \cdot Y) + (\bar{X} \cdot Z)$$

Duality of the above expression

$$\bar{X} + (Y \cdot Z) = (X + Y) \cdot (\bar{X} + Z)$$

3. Explain the following logic gats and show their functions by using a truth table.

AND GATE

The AND Gate has two or more input signals but only one out signal. The functions of AND gate are shown in the truth table given below.

INPUT		OUTPUT
X	Y1	X.Y
0	0	0
0	1	0
1	0	0
1	1	1

OR GATE

The OR Gate has two or more input signals. The functions of OR gate are shown in the truth table given below.

INPUT		OUTPUT
X	Y1	X+Y
0	0	0
0	1	1
1	0	1
1	1	1

NOT GATE

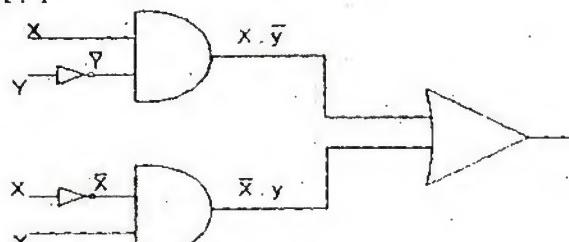
The NOT Gate is an inverter gate with only one input signal and one output signal. The function of AND gate are shown in the truth table given below.

INPUT		OUTPUT
X	X	X
0		1
1		0

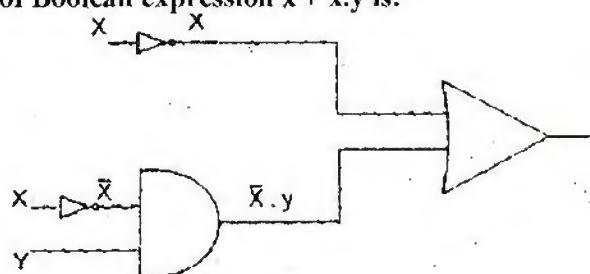
4. Represent the following Boolean expression as a combination of logic gates.

(a)

$$X \cdot \bar{Y} + \bar{X} \cdot Y$$



Logic circuit diagram of Boolean expression $x + x \cdot y$ is:



5. Simplify the following Boolean function using k-maps

Simplify the Boolean function $f(x, y) = x + x \cdot y$

The following steps can be performed to simplify this function using K-map.

Step 1: Represent the function as sum of Minterm form

$$\begin{aligned} f(x, y) &= x + x \cdot y \\ &= x \cdot 1 + x \cdot y \\ &= x \cdot (y + y) + x \cdot y \quad (\text{by axiom } y + \bar{y} = 1) \\ &= x \cdot y + x \cdot y + x \cdot y \quad (\text{by distributive law}) \end{aligned}$$

Step 2: Draw all possible minterms with two variables

X / Y	\bar{Y}	Y
X	$\bar{X} \cdot \bar{Y}$	$\bar{X} \cdot Y$
X	$X \cdot \bar{Y}$	$X \cdot Y$

Step 3: For each minterms presents in the function mark 1 in the corresponding square and mark 0 in the remaining square.

X / Y	\bar{Y}	Y
X	0	1
X	1	1

Step 4: Mark a group of two or four adjacent 1

X / Y	\bar{Y}	Y
X	0	1
X	1	1

Step 5: Write simplified expression for each group

First Group:

$$\bar{X} \cdot \bar{Y} + X \cdot \bar{Y}$$

$$\bar{X} \cdot (Y + \bar{Y})$$

$$\bar{X} \cdot (1)$$

Second Group:

$$\bar{X} \cdot Y + X \cdot Y$$

$$Y \cdot (\bar{X} + X)$$

$$Y \cdot (1)$$

$$Y$$

Step 5: Write the final simplified form as a sum of products

$$f(X, Y) = X + Y$$

Simplify the Boolean function $f(x, y, z) = \bar{x} \cdot y \cdot z + x \cdot \bar{y} + x \cdot \bar{y} \cdot z$

The following steps can be performed to simplify this function using K-map.

Step 1: Represent the function as sum of Minterm form

$$\begin{aligned} f(x, y, z) &= \bar{x} \cdot y \cdot z + x \cdot \bar{y} + x \cdot \bar{y} \cdot z \\ &= \bar{x} \cdot y \cdot z + \bar{x} \cdot \bar{y} \cdot 1 + x \cdot \bar{y} \cdot z \\ &= \bar{x} \cdot y \cdot z + x \cdot \bar{y} \cdot (z + \bar{z}) + x \cdot \bar{y} \cdot z \quad (\text{by axiom } Z + \bar{Z} = 1) \\ &= x \cdot y \cdot z + x \cdot \bar{y} \cdot z + x \cdot \bar{y} \cdot z + x \cdot y \cdot z \quad (\text{by distributive law}) \end{aligned}$$

Step 2: Draw all possible minterms with two variables

X / Y	$\bar{Y}Z$	\bar{Y}, \bar{Z}	\bar{Y}, Z	Y, \bar{Z}
\bar{X}	$\bar{X}, \bar{Y}, \bar{Z}$	\bar{X}, \bar{Y}, Z	\bar{X}, Y, \bar{Z}	X, \bar{Y}, \bar{Z}
X	X, \bar{Y}, Z	X, \bar{Y}, \bar{Z}	X, Y, \bar{Z}	X, Y, Z

Step 3: For each minterms presents in the function mark 1 in the corresponding square and mark 0 in the remaining square.

X / Y	$\bar{Y}Z$	\bar{Y}, \bar{Z}	\bar{Y}, Z	Y, \bar{Z}
\bar{X}	0	1	1	0
X	1	1	0	0

Step 4: Mark a group of two or four adjacent 1

X / Y	$\bar{Y}Z$	\bar{Y}, \bar{Z}	\bar{Y}, Z	Y, \bar{Z}	Y, Z
\bar{X}	0	1	1	1	0
X	1	1	0	0	0

Step 5: Write simplified expression for each group

First Group:

$$\bar{X}Y, Z + X, \bar{Y}, Z$$

$$\bar{Y}, Z, (\bar{X} + X)$$

$$\bar{Y}, Z, (1)$$

$$\bar{Y}, Z$$

Second Group:

$$\bar{X}, \bar{Y}, Z + X, \bar{Y}, Z$$

$$\bar{X}, Y, (Z + Z)$$

$$\bar{X}, Y, (1)$$

$$\bar{X}, Y$$

Third Group:

$$\bar{X}X, \bar{Y}, Z + X, \bar{Y}, Z$$

$$\bar{Y}, Z, (\bar{X} + X)$$

$$\bar{Y}, Z, (1)$$

$$\bar{Y}, Z$$

Step 5: Write the final simplified form as a sum of products

$$\bar{Y}, Z + X, \bar{Y} + \bar{Y}, Z$$